A Method for Reconciling Values of Parameters

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ABSTRACT

Adjusting a set of values in order to conform to a set of requirements or relations usually involve subjective judgment and reconciliation. Consider the case of budget preparation, in which the total available fund is set. Most often, the sum of the requested amounts from different concerned parties exceeds the total available budget. Adjusting the individual requests requires negotiation and political maneuvering, and often the "losers" and the "winners" emerge. This paper presents a method that allocates the resource among the concerned parties in a rational and agreeable manner reflecting the degree of desires of the individual parties. A fuzzy set is used to represent the notion of desire, and the fuzzy optimization approach is used to find the most "acceptable" set of values. Mathematically, the approach reduces to a linear programming if the requirements are in the linear form. This process can be made iterative so that the elasticity of individual "desire" can be re-negotiated; in other words, change of mind is accommodated. The proposed approach can be used for various adjustment situations, both subjective resource allocation (money, space, manpower, and time) and also adjustment of physical quantities, e.g. measurement of distances, and weights, to achieve consistency.

1. INTRODUCTION

Imagine a situation in which an analyst is preparing the budget for a project. Usually the sum of the funds required for different needs does not match (usually exceeds) the total available funds; and hence, the difference must be reconciled. The process of reconciling individual values requires various negotiations among the parties that may be impacted by the adjustment.

Similar situations are found frequently, when adjusting the values of design parameters, examining the consistency of the observed values in relation to underlying requirements among them, and calibrating the values of parameters of a formula. This paper proposes a method that helps to reconcile the individual values so that a desired consistency is achieved for the set of the values.

2. NATURE OF THE PROBLEM AND MODEL REQUIREMENTS

The problem usually has the following characteristics.

The problem. A set of values are available. They are supposed to be related by a certain set of relationships, but the values do not meet the relationships. The problem is to adjust the values so that they meet the relationships.

The values given initially and to be adjusted may be observed (or measured) values, estimated values, or desired values, and uncertainty is associated with each value with respect to the acceptability and tolerance.

The relationships that the values must meet may be a “hard” equality or inequality, or a “soft” equality or inequality. The relationships may be (1) they satisfy a set of equality (exact or approximate) relationships; or (2) they satisfy a set of approximate inequality relationships (e.g. "much greater than," etc.).

The purposes of adjustment. The following situations may require the analyst to adjust the values.

-When the values are used as input to another set of models, or an input to a computer package.

-When the analyst wishes to change one or more of the already agreed upon design values to reflect his/her or the client’s desire for design modification.

-When the data points are known, the parameters of an equation that fit the data points best need to be found.

Model requirement. The method needs to meet the following general requirements:

1. The final adjusted value should be close to the initially presented value as much as possible, and they should meet the satisfaction or desire of the analyst or the concerned party(ies).

2. The method allows the opportunity for the analyst to interact with those who are directly involved in deciding on the acceptability of the values, and also allow dialogues and readjustment.

3. The method should have the capacity to measure the level of satisfaction (or acceptability) of the adjusted values.

3. TRADITIONAL APPROACHES AND THEIR LIMITS

Traditionally, this type of problem has been handled in two ways: (1) by the trial-and- error method, in which an approximate band of tolerable deviation is assumed for each value. Within the band, each value is adjusted by trial-and-error (sometimes involving negotiation); and (2) by the least- squared method, in which the squared sum of the differences between the initial values and the adjusted values is minimized.

These approaches cannot accommodate the soft notion of "desire" and "tolerance" in the mind of the analyst or the concerned party in a systematic manner. Such a method is particularly useful for the early stage of planning or design in which many parameters of decision and design are uncertain to the analyst (or even to the concerned parties).

4. THE PROPOSED METHOD

Treatment of Individual Values and Their Mathematical Representation

The initial value harbors the notion of desire, approximation, or tolerance. A fuzzy number is introduced to represent these notions. The membership function of a fuzzy set A, A(x), characterizes the degree of "compatibility," or "satisfaction" between a value, x, with the notion that fuzzy set A represent, by a value between 0 and 1: $A(x)=1$, when the value of x is considered most compatible with notion A.

Treatment of Required Relationships and their Mathematical Representation

The relationships that the values must satisfy may be a combination of equality and inequality both in the exact and the non-exact sense.

The "hard" equality or inequality relationships are usually associated with the physical principles, such as the conservation of flow in which the total input must be equal to the total output.

The "soft" equality and inequality relations arise when the analyst is not certain about the exact relationship. An approximate relation, such as "x is 'much' greater than y," and "x and y are 'approximately' equal," are the fuzzy relations.

The Optimization Model: Concept

The problem described above is an optimization process, in which the individual values wish to "stick" to the initial values, but they are "pulled" from them in order to meet the required relations. Thus, the solution is found at the point of compromise.

Such an optimization process follows the Bellman-Zadeh principle; that is, the solutions, or the decision set, lies in the confluence of the goal and the constraints: $D=C\cap G$, where D is the solution set, C being the constraint set, and G being the goal set. The solution set, D, is a set of values that satisfies both C and G. The objective of optimization is to select a value that "best" satisfies both C and G.

The process can be formalized as

$$D(x^*) = \text{Max}\ \{\text{Min}\ [C(x), G(x)]\},\ \text{for } x \in X, \qquad (1)$$

where x* is the best solution. D(x*) indicates the degree of overall satisfaction of the best solution, and C(x) and G(x) indicate the degree that x satisfies the constraint and the goal, respectively.

Finding the value of x* in Eq.(1) is equivalent to solving the following optimization problem, where the unknowns are x and h.

$$\begin{aligned} &\text{Max } h \\ &\text{Subject to} \\ &\qquad C(x) \geq h, \text{ and } G(x) \geq h \\ &\qquad x \geq 0 \text{ and } h \geq 0. \end{aligned} \qquad (2)$$

The Optimization Model: Formulation

Assume the following,
w1, w2,…: the initial values, which the analyst (or stakeholder) wishes to adjust.
W1(x1), W2(x2)…: the membership function of the fuzzy numbers derived from the initial value.
$f_{R1}(x_1,x_2,x_3,..)$, $f_{R2}(x_1,x_2,x_3..)$,: "hard" relationships and requirements that the values must satisfy.
FR1 $(x_1,x_2,x_3..)$, FR2$(x_1,x_2,x_3..)$, the membership functions of the "soft (fuzzy)" relationships that the adjusted values must satisfy.

The general formulation of the optimization process is

$$\begin{aligned} &\text{Max } h \\ &\text{Subject to} \end{aligned} \qquad (3)$$

For initial values: W1 $(x_1) \geq h$, W2 $(x_2) \geq h, \ldots,$ (4)
For hard relations: $f_{R1}(x_1, x_2, x_3 \ldots)=0$, $f_{R2}(x_1, x_2, x_3, \ldots)=, >$ or $< 0, \ldots$ (5)
For soft relations: FR1$(x_1,x_2,\ldots) \geq h$, FR2$(x_1, x_2,\ldots) \geq h, \ldots.$ (6)
$x_1, x_2, \ldots \geq 0$, $h \geq 0$,

5. EXAMPLE

Initial values. Consider five values of the parameters need to be adjusted, X1, X2, X3, X4, and Z1. Their initial values are 450 ($=x_1^0$), 200 ($=x_2^0$), 120 ($=x_3^0$), 160 ($=x_4^0$), and 1100 ($=z_1^0$), respectively. The tolerance for deviation from the value is for X1, 450 ±100; for X2, 200 ± 80; for X3, 120 ± 50; for X4, 160 ± 90, and for Z1, 1100± 100. The corresponding membership functions are shown in the upper layer of Figure 1.

Relationships. Four relationships are considered.

Relationship 1: $x_1 + x_2 + x_3 + x_4 = z_1$, (7)
Relationship 2: x_1 is somewhat greater than (x_2+x_3),
with the acceptable difference no more than 50 (8)
Relationship 3: $x_3 \approx x_4$,(x_3 is approximately equal to x_4)
with the acceptable difference no more than 100 (9)
Relationship 4: $x_2 \approx 2x_3$, (x_2 is approximately twice of x_3)
with the acceptable difference no more than 50 (10)

LP Formulation

The objective is to maximize the value of h according to the formulation in Eq.(3).

The membership functions for parameter values, x_1, x_2, x_3, x_4 and Z_1 are defined as a triangular function. For the right- and left-hand side of the triangle, respectively; the corresponding equations are $h_{xi+}(x_i)$ and $h_{xi-}(x_i)$, respectively.

$$h_{xi+}(x_i) = -\frac{1}{RXi\,(orRZ_1)}(x_i - x_i^0) + 1 \geq h, \quad h_{xi-}(x_i) = \frac{1}{LXi\,(orLZ_1)}(x_i - x_i^0) + 1 \geq h, \quad \text{for } i=1,..4$$

where $x_1^0 = 450$, $x_2^0 = 200$, $x_3^0 = 120$, $x_4^0 = 160$, $z_1^0 = 1100$ and $RX_1 = LX_1 = 50$, $RX_2 = LX_2 = 40$, $RX_3 = LX_3 = 25$, $RX_4 = LX_4 = 40$, $RZ_1 = LZ_1 = 50$.

For the constraints based on the required relationships:

Relationship 1: $x_1+x_2+x_3+x_4=z_1$.

Relationship 2: $h_{\text{somewhat greater than zero}}(w_1) = -\frac{1}{RW_1} w_1 + 1 \geq h$ where $w_1 = x_1 - (x_2+x_3)$, and $RW_1 = 50$.

Relationship 3: $h_{\text{approximately zero+}}(w_3) = \frac{-1}{RW_3} w_3 + 1 \geq h$, $h_{\text{approximately zero -}}(w_3) = \frac{1}{LW_3} w_3 + 1 \geq h$

where $w_3 = x_3 - x_4$, and $RW_3 = LW_3 = 100$.

Relationship 4: $h_{\text{near equal to zero +}}(w_4) = \frac{-1}{RW_4} w_4 + 1 \geq h$ $h_{\text{near equal to zero -}}(w_4) = \frac{1}{LW_4} w_4 + 1 \geq h$

where $w_4 = x_2 - 2x_3$, and $RW_4 = LW_4 = 50$.

The above formulation is summarized in the linear programming format as below.

Max h
Subject to
Constraints representing parameter values:

	Right- hand side	Left- hand side
x1:	$-100h+x1\geq350$	$100h+x1 \leq550$
x2:	$-80h+x2\geq120$	$80h+x2 \leq280$
x3:	$-50h+x3 \geq70$	$50h+x3 \leq170$
x4:	$-90h+x4 \geq70$	$90h+x4 \leq250$
z1:	$-100h+ z1 \geq1000$	$100h+ z \leq1200$

Constraints related to relationships
Relationship 1: $x1+x2+x3+x4-z1=0$
Relationship 2 for the left-hand side $x1-x2-x3 \geq 0$ for the right-hand side $x1-x2-x3+50h \leq 50$
Relationship 3 for the left-hand side $x3-x4+100h \leq 100$ for the right-hand side $x3-x4-100h \geq -100$
Relationship 4 for the left-hand side $x2-2x3+50h \leq 50$ for the right-hand side $x2-2x3-50h \geq -50$

X1, x2, x3, x4, z1, and $h \geq0$

The Solution

Applying linear programming the adjusted values, the satisfaction level relative to the initial value, and also the satisfaction level relative to the required relationships are obtained as below.

With respect to the parameters to be adjusted,

Parameter	Initial value (range)	Adjusted value	Satisfaction (h-value)
X1	450 (±100)	428	0.78
X2	200 (± 80)	252	0.35
X3	120 (± 50)	142	0.56
X4	160 (± 90)	209	0.45
Z1	1100 (± 100)	1031	0.33* (min. satisfaction)

*: Max-Min solution of h.

With respect to the relationships,

Relationships	With the adjusted values	Satisfaction h- value
1. $x_1+x_2+x_3+x_4=z_1$	428+252+142+209 = 1031	1.0
2 x_1 is close somewhat greater or equal to (x_2+x_3)	428 is close but somewhat greater or equal to [252+142] (=396)	0.34
3 $x_3 \approx x_4$	$142 \approx 209$	0.34
4. $x_2 \approx 2x_3$	$252 \approx 2 \times 142$ (=284)	0.34

Summary

The above optimization problem may or may not yield a solution. When the solution is found, the value of h indicates the degree that all the constraints and the desire for the initial values are minimally satisfied. The satisfaction of individual relationships can be found by introducing the derived x_i's to respective membership functions, Eqs.(5) and (6). If the analyst requires that the solution must meet at least a given level of satisfaction, α, then an additional constraint, $h \geq \alpha$, can be added as a constraint.

If the solution does not exist, then a set of values that satisfies the initial desire for the parameters and also the required relationships does not exist. A possible action is to revise some (or all) of the initial values, or to change the membership function of the initial values and/or fuzzy relations by widening its base or shifting the base. The readjustment of the membership functions may involve negotiation among the concerned parties.

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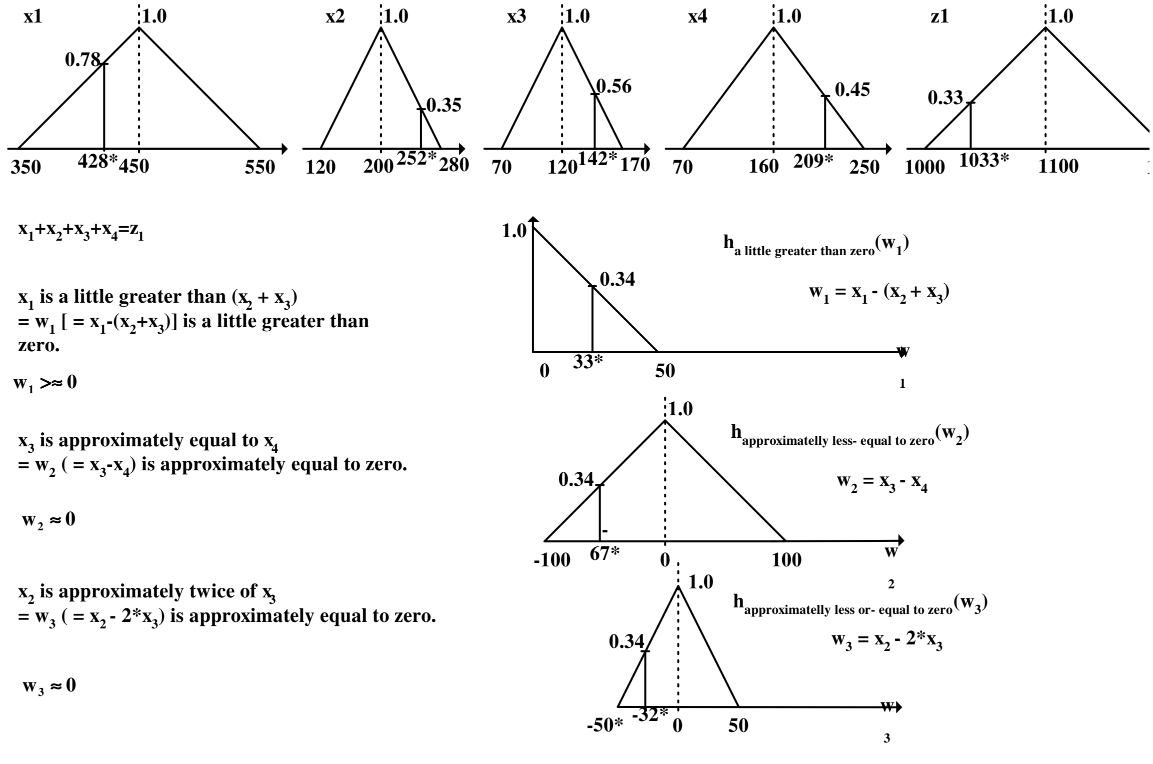

Figure 1. Illustrations for Solutions of the Example Problem